GW'S TOWARDS FUNDAMENTAL PRINCIPLES OF GR

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ABSTRACT. The physical non-existence of gravitational waves (GW's) as a consequence of the non-existence in general relativity (GR) of physically privileged reference frames, and of the "plasticity" of relativistic notion of a coordinate system.

1. – In 1944 Weyl proved that the *linearized* approximation of GR is **inadequate** to an appropriate treatment of the so-called GW's [1].

Moreover, in the *exact* (non-approximate) formulation of GR there exist various demonstrations of the *physical* non-existence of GW's [2].

In this Note I give another proof that the GW's are only mathematical undulations quite destitute of a physical reality. The argument is composed of two propositions, that have been already sketched by me in different contexts. However, for its significance, the question deserves a more complete illustration.

The following proof is qualitative and very simple. It is a straightforward consequence of the freedom of choice of the reference systems and of their "plasticity" [3].

2. – In the *exact* GR we have no class of physically privileged reference systems. Now, consider the spacetime manifold V connected with a celestial body B, and an observer Ω who is at rest together with B; Ω does not register any gravitational undulation emitted by B. (In Maxwell theory an inertial observer I, who is moving together with a charge C, does not register any e.m. wave emitted by C). However, the same conclusion holds also for a distant observer Ω', who is in motion with respect to Ω/B: for him too, B does not send forth any GW. (In Maxwell theory an inertial system I', who is in motion with respect to I/C, does not register any e.m. wave emitted by C). Indeed, if Ω' found that B has emitted a GW, we could affirm that reference Ω' has a privilege which is not shared by reference Ω, *i.e.* that Ω' belongs to a privileged class of reference frames.

Furthermore, as it was remarked by Weyl [3], "... *der Begriff der Relativbewegung zweier Körper gegeneinander in der allgemeinen Relativitätstheorie ebensowenig einen Sinn hat wie der Begriff der absoluten Bewegung eines einzigen.* Solange man noch den starren Bezugskörper zur Verfügung hatte

und an die Objektivität der Gleichzeitigkeit glauben konnte, auf dem Stand-
punkte Machs etwa, unter der Herrschaft der 'kinematischen Gruppe' gab
es eine relative Bewegung; aber in der allgemeinen Relativitätstheorie hat
sich das Koordinatensystem so 'erweicht', daß auch davon nicht mehr die
Rede sein kann. Wie die beiden Körper sich auch bewegen mögen, immer
kann ich durch Einführung eines gleigneten Koordinatensystems sie beide
zusammen auf Ruhe transformieren." *I.e.*: in GR the notion of reference
frame has acquired a characteristic "plasticity", so that it is always possible
to choose a coordinate system for which, *e.g.*, both Ω/B and Ω' are *at rest*.
Another application of Weyl's remark: the orbital motions of the stars of
a binary system can be "obliterated" with a suitable choice of the coordi-
nates – with the obvious conclusion that their revolution motions cannot
give out any GW; *a conspicuous instance is represented by the famous bi-
nary* B PSR1913+16. (Clearly, the reductions to rest are possible owing to
the "transport, or inertial, forces" associated with the convenient coordinate
frames).

3. – It is instructive to recall that similar, but less general, results exploiting
the "plasticity" of the coordinate systems had been obtained in 1924 by
Hilbert [4] and in the Sixties of past century by Landau and Lifchitz [5].

Hilbert emphasizes that in the problem of the Einsteinian field of a gra-
vitating body (extended or point-like) of a given mass M, we can choose a
coordinate system for which a moving test-particle and the gravitating body
are *both* reduced to rest. He gives a simple example of this fact. To be de-
terminate and physically significant, assume that our gravitating body has
the minimal radius $r_{min} = (9/8)(2m)$, where $m \equiv GM/c^2$. Let us consider
a circular motion on an orbit $r = $ constant. Hilbert proves that this motion
is *restricted* by the following conditions (Hilbert puts $c = 1$; his α is equal
to $2m$):

$$(1) \qquad\qquad r > \frac{3}{2}(2m) > \frac{9}{8}(2m) \quad ,$$

$$(2) \qquad\qquad v < \frac{1}{\sqrt{3}} \quad ,$$

where $v = (m/r)^{1/2}$ is the linear velocity.

For $r = r_0$, say, the angular velocity $(\mathrm{d}\varphi/\mathrm{d}t)_0$ is given by

$$(3) \qquad\qquad \left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)_0 = \left(\frac{m}{r_0^3}\right)^{1/2} \quad .$$

If we consider a co-rotating reference system, the centre of which coincides
with the centre of the gravitating body, we have:

$$(4) \qquad\qquad \varphi \to \varphi + \left(\frac{m}{r_0^3}\right)^{1/2} t \quad ;$$

thus, the spacetime interval

(5) $$\frac{r}{r-2m}\,dr^2 + r^2\,d\varphi^2 - \frac{r-2m}{r}\,dt^2$$

is transformed into

(6) $$\frac{r}{r-2m}\,dr^2 + r^2\,d\varphi^2 + 2\left(\frac{m}{r_0^3}\right)^{1/2} r^2 d\varphi\,dt + \left(\frac{m}{r_0^3}r^2 - \frac{r-2m}{r}\right)dt^2 \quad .$$

For $r = r_0$, we have:

(7) $$\frac{r_0}{r_0-2m}\,dr^2 + r_0^2\,d\varphi^2 + 2\,(m\cdot r_0)^{1/2}\,d\varphi\,dt + \left(\frac{3m}{r_0}-1\right)dt^2 \quad ,$$

and since $r_0 > (3/2)(2m)$, the *pseudo*-Riemannnian property of the interval is preserved, *i.e.* transformation (4) is an appropriate spatiotemporal transformation, which reduces actually to rest *both* the test-particle and the gravitating body.

Of course, Hilbert adopts the *standard* form of solution to Schwarzschild problem, which was discovered by him, by Droste, and by Weyl, independently. And it is very interesting that inequality (1) is stronger than inequality $r > 2m$, which is the *validity condition* of the standard form – as it was prescribed by its discoverers.

The inequalities (1) and (2) are quite "anti-Newtonian": they are a consequence of the fact that, for suitably small values of r, the spacetime curvature exerts on the test-particle a **repulsive** action. And we see that the pseudo-Riemaniann character of interval (7) is closely connected with this gravitational repulsion.

3bis. – Landau and Lifchitz [5] observed that the maximal number of test-particles moving in a given gravitational field, which can be reduced to rest by means of a convenient choice of coordinates, is *four*. Precisely, if we assume that the particles are situated at the four vertices of a tetraedron (which has six edges), it is possible to find a coordinate system such that they are at rest. Obviously, the choice of a tetraedron is not casual; in particular, just six are the essential components of the metric tensor.

3ter. – The world-line of any test-particle P in a given gravitational field is a *geodesic* line, and therefore no emission of GW's is possible. The reduction to rest of P yields a reinforcement of this conclusion.

4. – GR and Maxwell e.m. theory are basically different: in Maxwell theory the inertial frames are physically privileged, and the accelerated charges emit e.m. waves; in GR, on the contrary, *no* reference system is physically privileged, and the accelerated masses do not emit GW's.

The formal (and partial) resemblance of the *linear* version of GR with the e.m. theory has deceived many physicists. Unfortunately, the important paper by Weyl of 1944 [1] is ignored in the current literature.

5. – Another consequence of the "plasticity" of the coordinate systems is the following (cf. [6]). Let us consider a swarm of corpuscles, gravitationally

and non-gravitationally interacting, and fix our attention upon the world-line L of any of them. By virtue of a beautiful and well-known theorem by Fermi, we can choose a coordinate system for which the metric tensor is *constant* on the whole line L. This means that the gravitational field *on L* has been blotted out. Consequently, no GW has been emitted by our corpuscle. Now, an identical argument can be applied to the other corpuscles, singularly taken into consideration. (If the corpuscles interact only gravitationally, their world-lines are *geodesic* lines).

6. – A last and impressive consequence of the "plasticity" of the coordinate systems in GR: it is easy to see that the *undulatory character* of any wavy solution of Einstein field equations can be crossed out by a suitable change of the coordinates. The result is a metric tensor which is generally non-stationary.

APPENDIX

In February 2007 and in March 2007 the LIGO Scientific Collaboration published two papers, resp. entitled: "Upper limits on gravitational wave emission from 78 radio pulsars" [7], and "Search for gravitational wave radiation associated with the pulsating tail of the SGR 1806-20 hyperflare of 27 December 2004 using LIGO" [8].

The apparatuses did not register any GW.

A comment. The LIGO researchers have an unjustified belief in the validity of the obsolete theory of GW's, which is derived from the linearized approximation of GR, and in the physical adequacy of the various undulatory solutions of Einstein field equations (see, *e.g.* [9]), which have a mere formal character, *i.e.* a character devoid of physical significance [2]. These unfounded convictions induce them to persevere in the chase of a non-existing phenomenon: the GW's.

(In 1977 D. Kennefick wrote a review article entitled "Controversies in the History of the Radiation-Reaction problem in General Relativity" [10], which is a good testimony of the widespread conceptual confusion regarding the GW's. I am grateful to Prof. G. Morpurgo, who has called my attention on Kennefick's paper.)

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